Extension 2 Endogenous search effort

- Purpose: Derive optimal search strategies for employed job seekers in terms of a reservation wage conditional on unemployment benefits and labour market conditions.
- Basic assumptions:
  - Basically similar to the basic model with important exceptions:
  - Assume that the job offer arrival rate depends on the worker's search effort, in a increasing but diminishing way, i.e., $\lambda = \lambda(e)$, $\lambda'(e)>0$ and $\lambda''(e)<0$, where e denotes search effort,
  - Search effort is costly, in an convex way, i.e., $C(e)$, $C'(e)>0$, and $C''(e)>0$, where C expresses search cost and e search effort
  - Thus the instantaneous utility of a job seeker is $b-C(e)$.
  - For simplicity and exposition: No job-to-job mobility.

Extension 2 Endogenous search effort

- Remember Equation 5) from the previous lecture expressing and implicitly defining the reservation wage:

$$6) \quad x = b - C(e) + \frac{\lambda(e)}{r+q}\int_x^\infty (w-x)\,dH(w)$$

- Since an optimal search strategy is to accept a job offer if $V_e(w)>V_u$ and the unique reservation wage equals $x=r\,V_u$, we can apply standard maximization techniques. Criteria: $\partial x/\partial e=0$ as usual!
- This effort level will maximize the intertemporal utility of an unemployed job-seeker.
- Thus, by differentiating we find the optimum level of effort:

$$7) \quad C'(e) = \frac{\lambda'(e)}{r+q}\int_x^\infty (w-x)\,dH(w)$$

Extension 2 Endogenous search effort

- By using the optimum expression 7) inserted into 6) we get

$$8)\quad x = b - C(e) + \frac{\lambda(e)}{r+q} = \int_x^\infty (w-x)\,dH(w)$$

$$= b - C(e) + \frac{\lambda(e)}{r+q}\frac{C'(e)(r+q)}{\lambda'(e)} = b - \frac{\lambda(e)}{\lambda'(e)}C'(e) - C(e)$$

- The optimum condition 7) and Equation 8) comprise a system of equations, which implicitly define the reservation wage and the optimal effort as functions of the level of unemployment benefit. Let x(b) and e(b) denote the reservation wage and the optimal effort, respectively.

- How do x(b) and e(b) depend on b? (see assignment/seminar for details)
- Strategy: Start by differentiate 7) w.r.t. b.

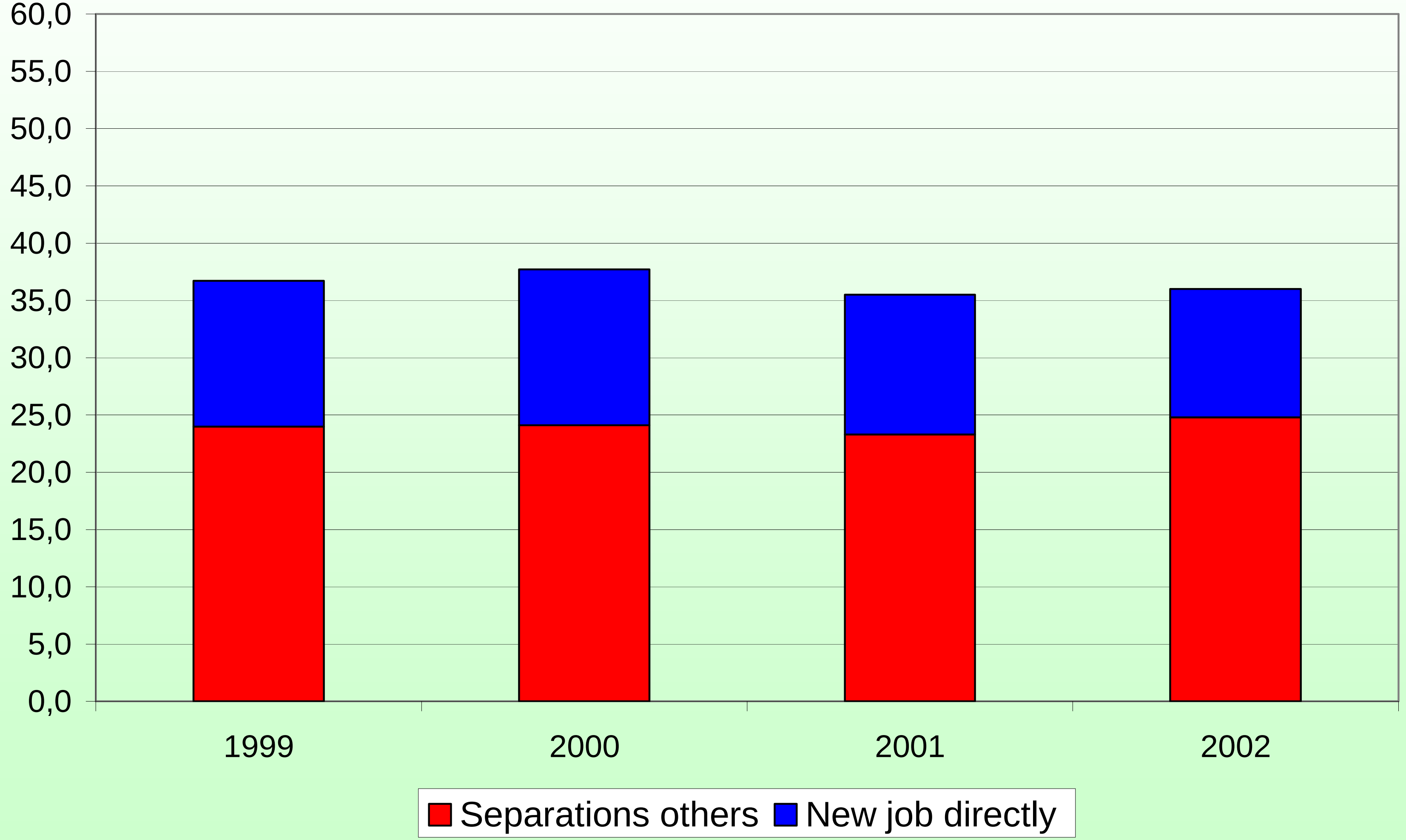
Extension 2 Endogenous search effort

- This will reveal that $\text{sign}(\partial x(b)/\partial b) \neq \text{sign}(\partial e(b)/\partial b)$
- Then differentiate 8) w.r.t. b. Since $\text{sign}(\partial x(b)/\partial b) \neq \text{sign}(\partial e(b)/\partial b)$ we find that $\partial x(b)/\partial b>0$ AND $\partial e(b)/\partial b<0$.
- So increased unemployment benefits yields with endogenous search (as before with exogenous job offer arrival rate) that the reservation wage increases AND that the search effort drops.
- WHY? Increased benefits increases the intertemporal utility as unemployed, and the jos seeker can search less intensively since the marginal gain from search effort (by getting a job) drops below the marginal disutility search effort provokes.
- Labour market conditions (e.g., recessions) can be analysed introducing $\lambda = \alpha\lambda(e)$, where α give info on the state of the labour market (see assignment/seminar). Differentiating of 7) and 8) show that $\text{sign}(\partial x(\alpha,b)/\partial\alpha) = \text{sign}(\partial e(\alpha,b)/\partial\alpha)$ we find that $\partial x(\alpha,b)/\partial\alpha>0$ AND $\partial e(\alpha,b)/\partial\alpha >0$

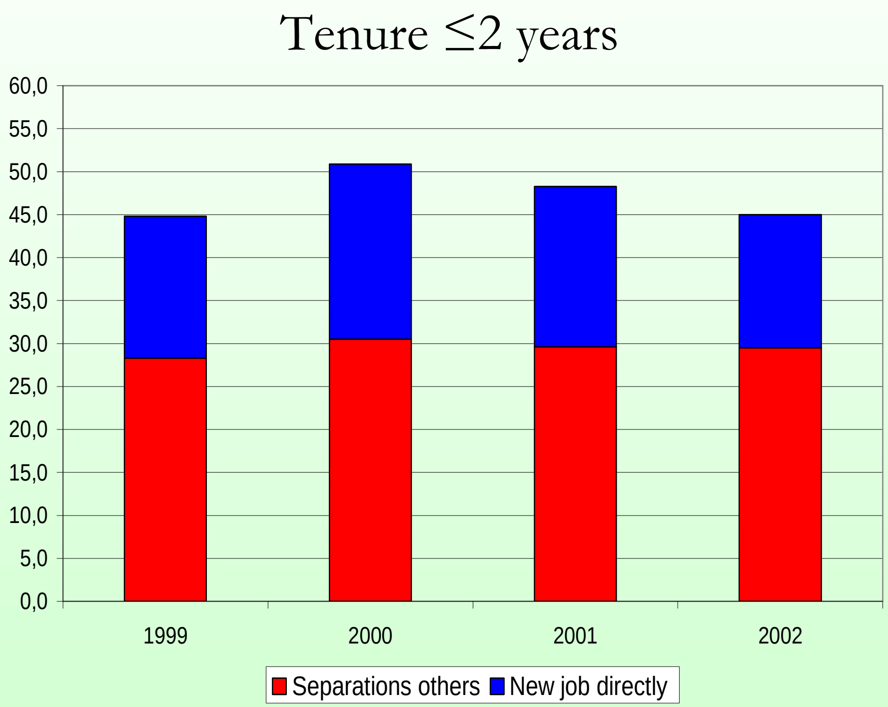
Extension 3 On the job search

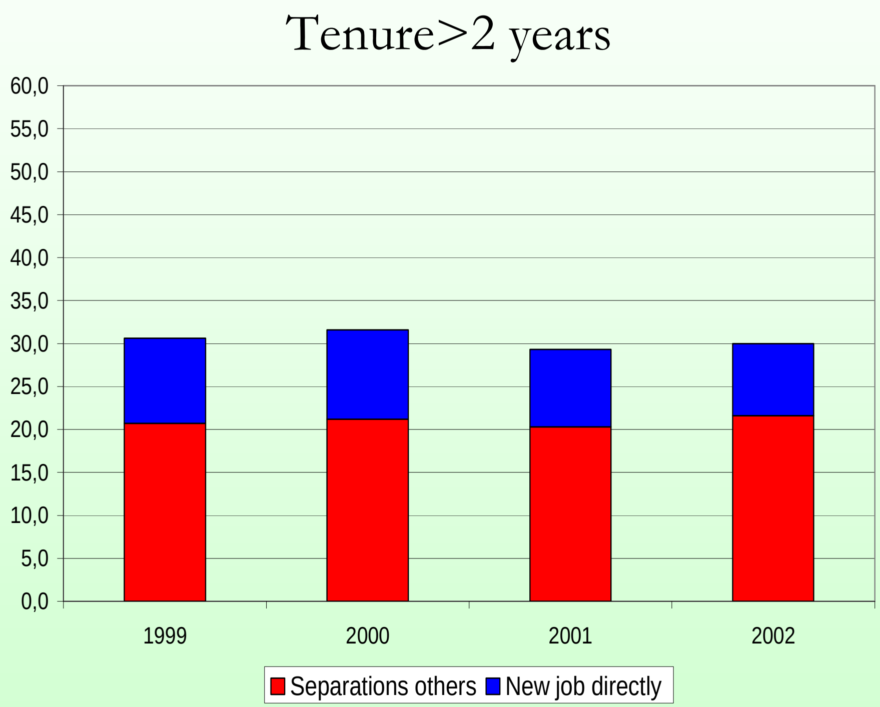
- Highly unlikely that only unemployed search for jobs. Employed workers search for better jobs!
    - Employed workers may be considered better than unemployed job-seekers (signalling)
    - Employed workers may have access to better search facilities.
    - Newly employed workers may gain access to job ladders and improved career opportunities.

Job-separations and job-to-job separations



Job-separations and job-to-job separations





Extension 3 On the job search

- Purpose: Derive an optimal search strategy for employed job seekers in terms of a reservation wage
- Basic assumptions:
  - Basically similar to the basic model with important exceptions:
  - Let the search costs of employed workers be negligible, search for these workers are assumed cost free. Let the job offer arrival rates of unemployed job seekers and employed workers be denoted by λ_u and λ_e, respectively,
  - Let pay during unemployment, z, be equal to unemployment benefits, b, less search cost, c (z=b-c).

Extension 3 On the job search

- The discounted expected utility of an employed worker currently earning w comprise of three elements:
  - An instantaneous income w from his or hers waged labour,
  - The average discounted expected gain (a loss really) of $q[V_u - V_e(w)]$ due to job loss (remember q expresses the job destruction rate),
  - The discounted expected earnings consequent upon a change of employer (which occurs for every wage exceeding the current wage w):

$$\lambda_e \int_w^{\infty} [V_e(\xi) - V_e(w)] H(\xi)$$

Extension 3 On the job search

- The discounted expected utility of an employed worker currently earning w:

$$9) \qquad rV_e(w) = w + q[V_u - V_e(w)] + \lambda_e \int_w^\infty [V_e(\xi) - V_e(w)]H(\xi)$$

- Note that $V_e(w)$ is increasing in w (see assignment/seminar for details). To see this, differentiate 9) while remembering Leibniz' rule, which then gives:

$$V_e'(w) = \frac{1}{r + q + \lambda_e[1 - H(w)]}$$

- Optimal search strategy for a job-seeker is to choose a reservation wage x such that $V_e(x) = V_u$.

Extension 3 On the job search

- The discounted expected utility of an unemployed worker:

$$rV_u = z + \lambda_u \int_x^\infty [V_e(\xi) - V_u] dH(\xi)$$

- Since $V_e(x) = V_u$ then x=w can be inserted into 9) which yields:

$$x + \underbrace{q[V_u - V_e(x)]}_{=0} + \lambda_e \int_x^\infty [V_e(\xi) - V_e(x)] dH(\xi) = z + \lambda_u \int_x^\infty [V_e(\xi) - V_u] dH(\xi)$$

- Thus we find that the reservation wage can be expressed:

$$10) \quad x = z + (\lambda_u - \lambda_e) \int_x^\infty [V_e(\xi) - V_u] dH(\xi)$$

thus the arrival rates (or difference) are crucial for the reservation wage!

Extension 3 On the job search

- How does the reservation wage then depends on the exogenous parameters/ variables (i.e., H(.), λ_u and λ_e) ?

- Note that $V_e(\xi)$- V_u is endogenous, but C&Z solve 10) as a function of H(.), λ_u and λ_e (see assignment/seminar for details).

$$x = z + (\lambda_u - \lambda_e)\int_x^\infty \frac{1 - H(\xi)}{r + q + \lambda_e[1 - H(\xi)]} d\xi$$

- Thus if λ_e =0 (no on-the-job search) back to basic solution.
- Thus if λ_e >0 future job opportunities reduce reservation wage.
- Thus if λ_e= λ_u then reservation wage become equal to net benefits.
- Thus if λ_e> λ_u then x may be less than z. Even a bad job provide better long-term opportunities than staying unemployed.

Equilibrium search models

- The cumulative wage distribution exogenous, equilibrium search models endogenize this distribution.

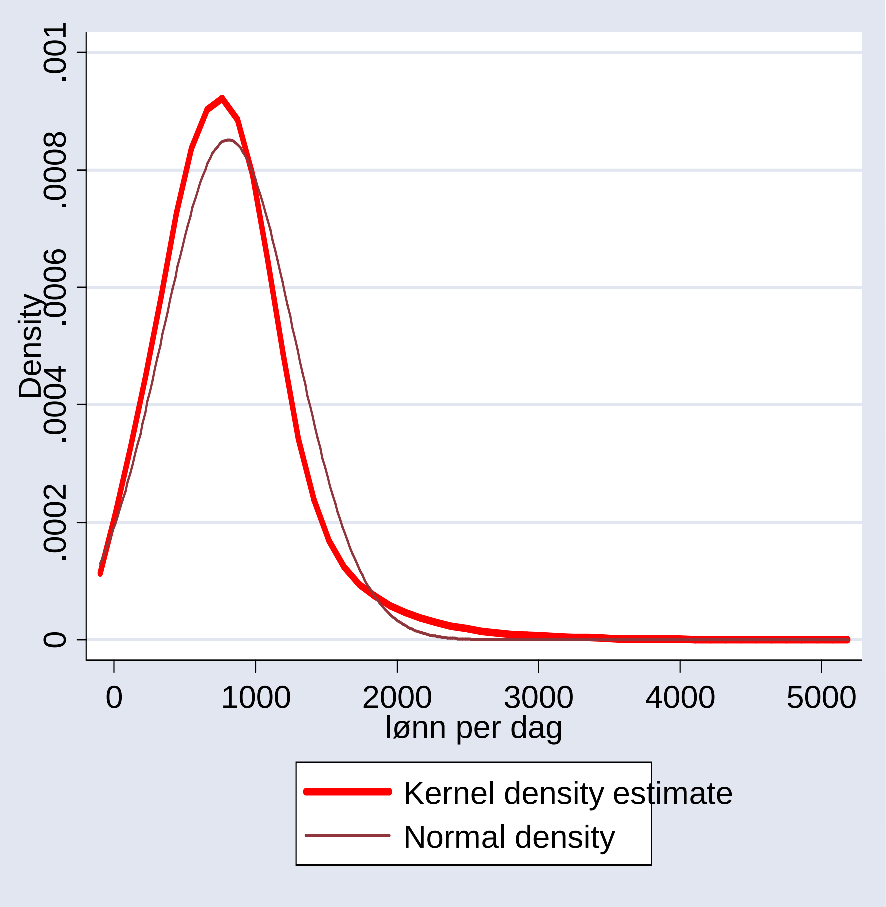
- Diamond's paradox: why does anyone pay above a reservation wage equal to x? Why do we observe wage dispersion in the economy?

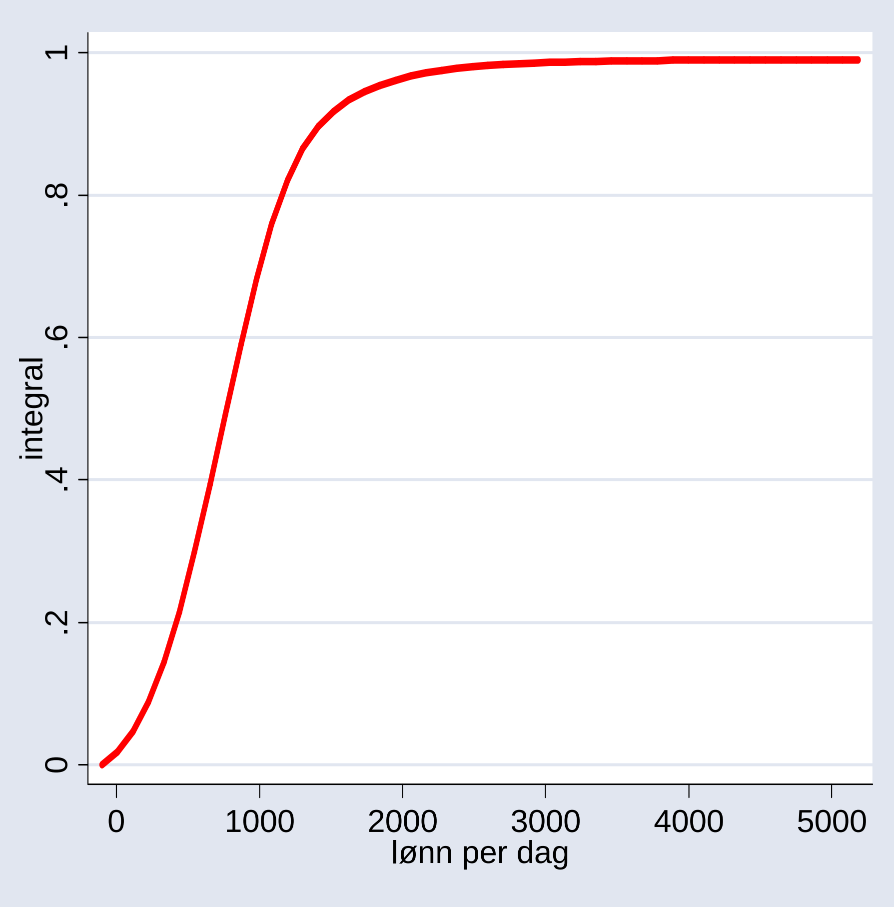
The basic job search model

- Assume that all possible wages (all that are offered) can be described by a probability distribution and this is known by all workers:

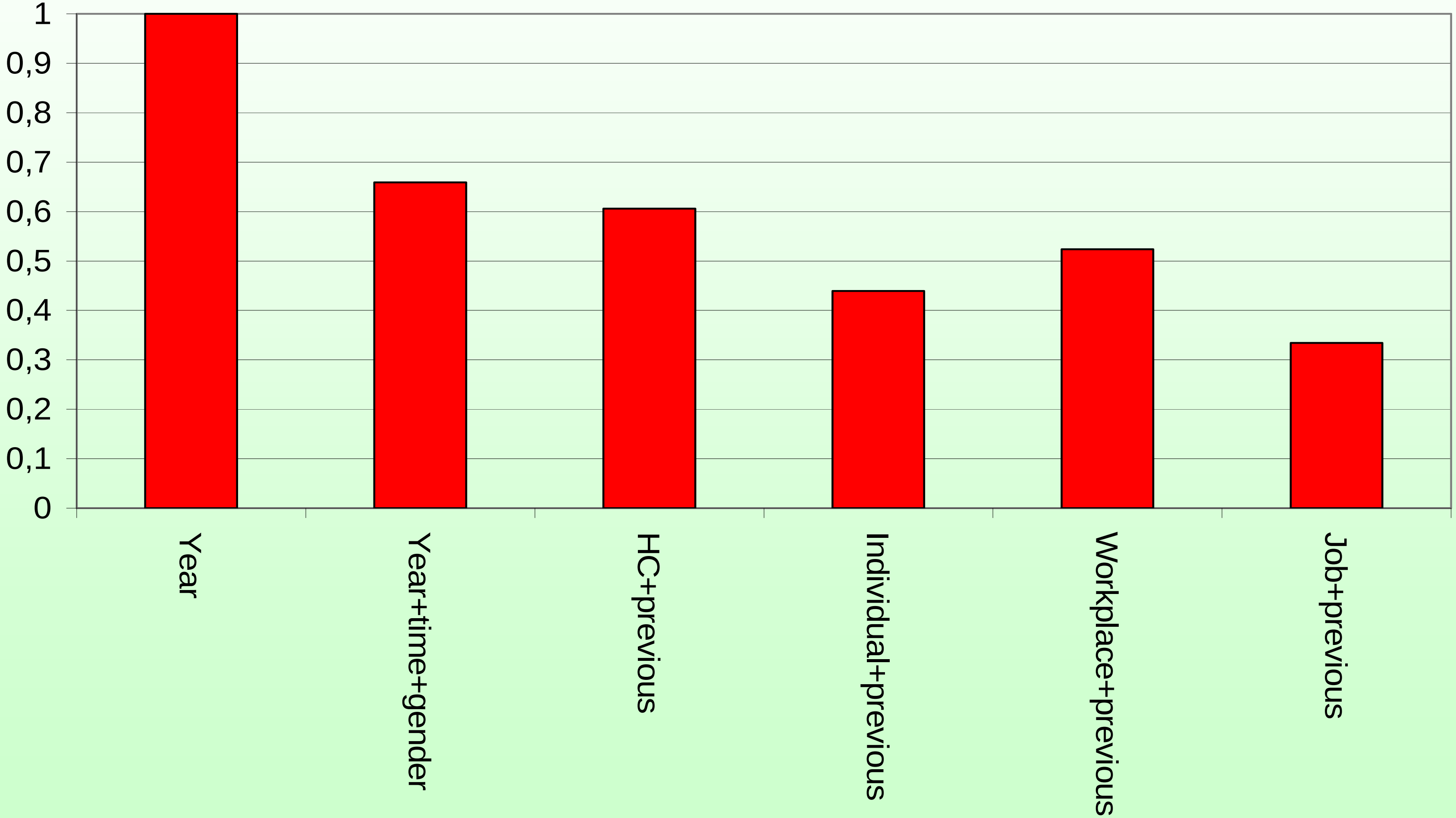
$$\Pr ob(W \leq w) = H(w) = \int_0^w h(w)dw$$

Wage distribution 2003 (1%-random sample)





Equilibrium search models

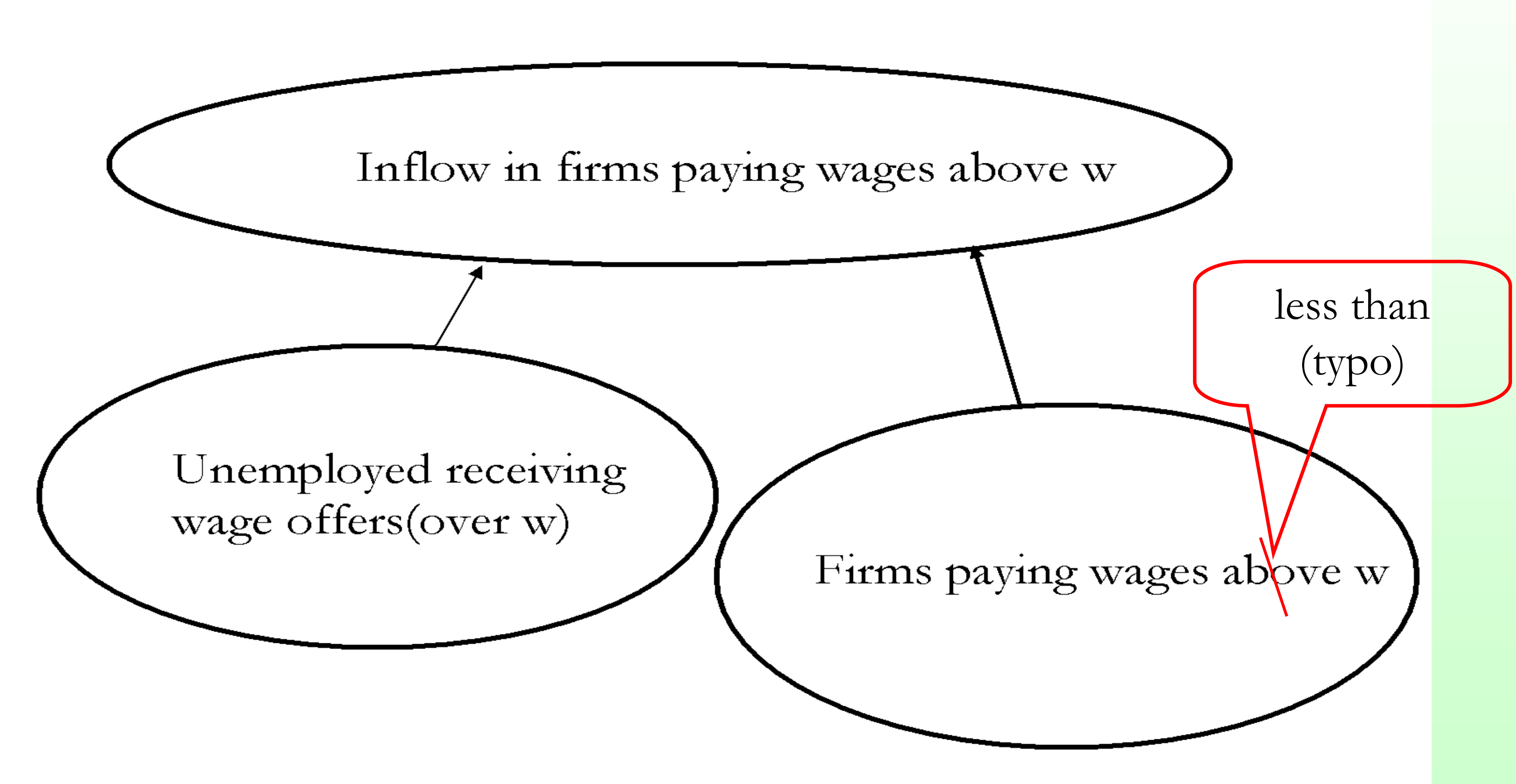


Equilibrium search models

- Basic assumptions:
  - Although the economy comprise of a continuum of workers(M) and a continuum of firms (F), these are normalised to unitary mass (1) (in the exposition) (alternatively you could assume that a factor M/F adjust appropriately the main equations).
  - Firms are profit maximizing, while workers maximize utility.
  - Let the job offer arrival rates of unemployed job seekers and employed workers be denoted by λ_u and λ_e, respectively. An exogenous job destruction rate is defined by q.
  - Let u denote the unemployment rate.
  - Let l(w) denote workforce size in a firm paying w (to all workers!). Let L(w) denote the aggregate employment of all firms paying a wage lower than w. Thus

$$L(w) = \int_0^w l(\xi)\, dH(\xi).$$

Equilibrium search models



Equilibrium search models

- Entry from unemployment: λ_u [1-H(w)]u
- Entry from firms paying w or less: λ_e [1-H(w)]L(w)
- Total entry in firms paying above w: [λ_u u + λ_e L(w)][1-H(w)]
- Employment in firms paying above w: 1-u-L(w)
- Exit from firms paying above w: q[1-u-L(w)]
- In a stationary equilibrium, exit equals entry, thus

$$[\lambda_u u + \lambda_e L(w)][1-H(w)]=q[1-u-L(w)]$$

- But since this must be true for each possible wage level, the derivate of lefthandside and righthandside must be equal.
- Since $L(w) = \int_0^w l(\xi)dH(\xi)$ then derivative then satisfy L'(w)=H'(w)l(w)

Equilibrium search models

- $\partial\{[\lambda_u u + \lambda_e L(w)][1\text{-}H(w)]\} / \partial w = \partial\{q[1\text{-}u\text{-}L(w)]\} / \partial w$
- $\partial\{\lambda_u u[1\text{-}H(w)]+\lambda_e L(w)[1\text{-}H(w)]\} / \partial w = \partial\{q\text{-}qu\text{-}qL(w)\} / \partial w$
- $-\lambda_u uH'(w)+ \lambda_e L'(w)[1\text{-}H(w)]-\lambda_e L(w)H'(w)=-qL'(w)$
- Plugging in L'(w)=H'(w)l(w) then yields:

  $-\lambda_u u\cancel{H'(w)}+ \lambda_e \cancel{H'(w)}l(w)[1\text{-}H(w)]-\lambda_e L(w)\cancel{H'(w)}=-q\cancel{H'(w)}l(w)$
- Rearranging and you get:

$$11)\qquad [q+\lambda_e[1-H(w)]]l(w)=\lambda_u u+\lambda_e\int_0^w l(\xi)dH(\xi)$$

- Since 11) is true for every wage, you can differentiate both sides (again), and find a function that implicitly defines all the functions l(.) and H(.) compatible with the flows in the equilibrium:

$$12)\qquad \frac{l'(w)}{l(w)}=\frac{2\lambda_e H'(w)}{q+\lambda_e[1-H(w)]}$$

Equilibrium search models

- Firm behaviour:
  - Each worker can produce an exogenous quantum y of goods.
  - The instantaneous profit received by the firm employing this worker is given by: $\Pi(w)=(y-w)l(w)$.
  - Ignore the interest rate, $r=0$, no discounting.
  - Max $\Pi(w)$ w.r.t. w give:

    13) $l'(w)/l(w)=1/(y-w),\ w\geq x$.

  - Note that the standard measure of monopsonistic power is given by $\varepsilon=w\,l'(w)/l(w)=w/(y-w)$, or, $(y-w)/w=1/\varepsilon$.

Equilibrium search models

- Equilibrium unemployment:
  - Flow into unemployment: $q(1-u)$.
  - Flow out of unemployment: $\lambda_u[1-H(x)]u= \lambda_u u$
  - Since in equilibrium the flow into and out of unemployment has to be equal, we find that the equilibrium unemployment rate is given by $u=q/(q+\lambda_u)$.

Equilibrium search models

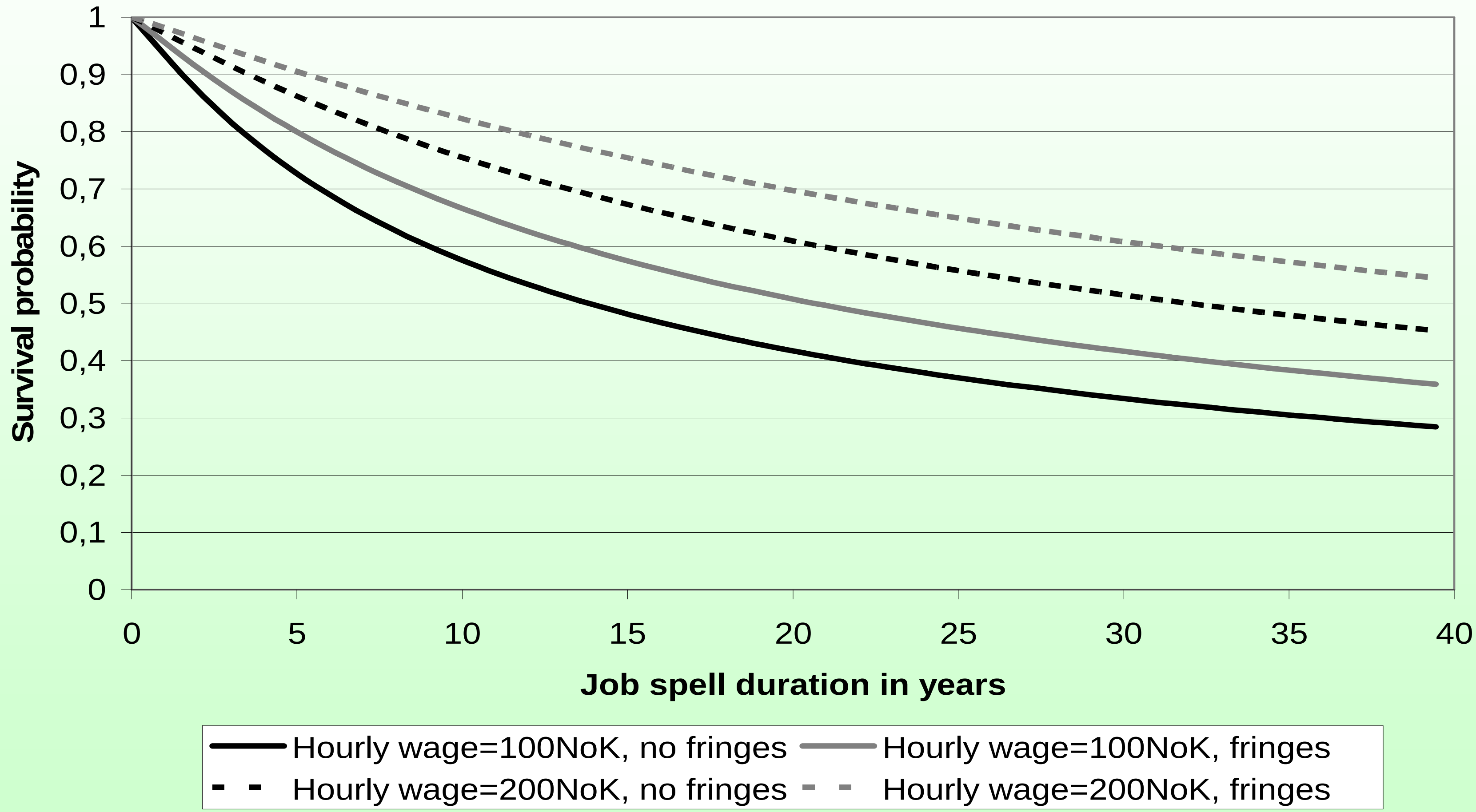
- The relationship between firm pay and firm size:
  - Since 13) $l'(w)/l(w)=1/(y-w)$, and $l'(w)/l(w)$ can represent the derivative of $\ln[l(w)]$ and since $\int[1/(y-w)]dw=-\ln[(y-w]$, we see that $\ln[l(w)]=-\ln[(y-w]+a$, where a is a constant.
  - Exponentiating give: $l(w)=A/(y-w)$, where $A=\exp(a)$.
  - If $w=x$ in Equation 11) then $l(x)= \lambda_u u/(q+\lambda_e)$.
  - Since $l(w)=A/(y-w)$ implies that $l(x)=A/(y-x)=\lambda_u u/(q+\lambda_e)$, thus $A=\lambda_u u(y-x)/(q+\lambda_e)$.
  - Since $u=q/(q+\lambda_u)$ in equilibrium, then we finally(!) find:

$$l(w) = \frac{q\lambda_u}{(q+\lambda_e)(q+\lambda_u)} \frac{(y-x)}{y-w)}, \quad l'(w) > 0.$$

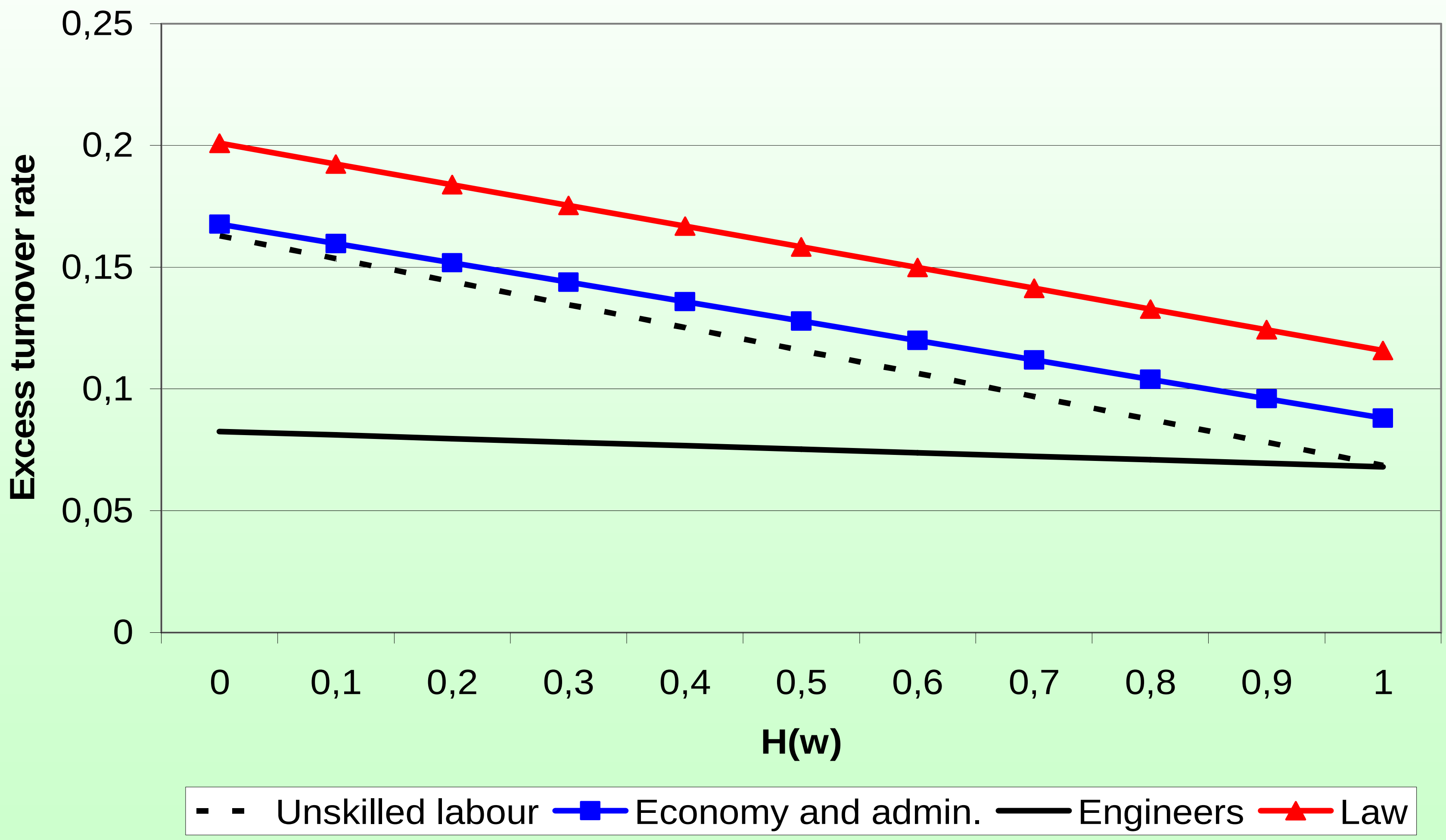
Equilibrium search models

- Implications:
  - The profit of each firm is equal in equilibrium.
  - Some firms achieve this paying low wages and employing few workers, other firms achieve this by paying high wages and succesfully employing many workers. Low-paying firms experience a high quit rate (causing small size), while high paying firms experience the opposite (thus large size)!

Equilibrium search models



Equilibrium search models



Barth and Dale-Olsen (2002)

Equilibrium search models

- Potential problem:
  - Consider the equilibrium wage distribution:

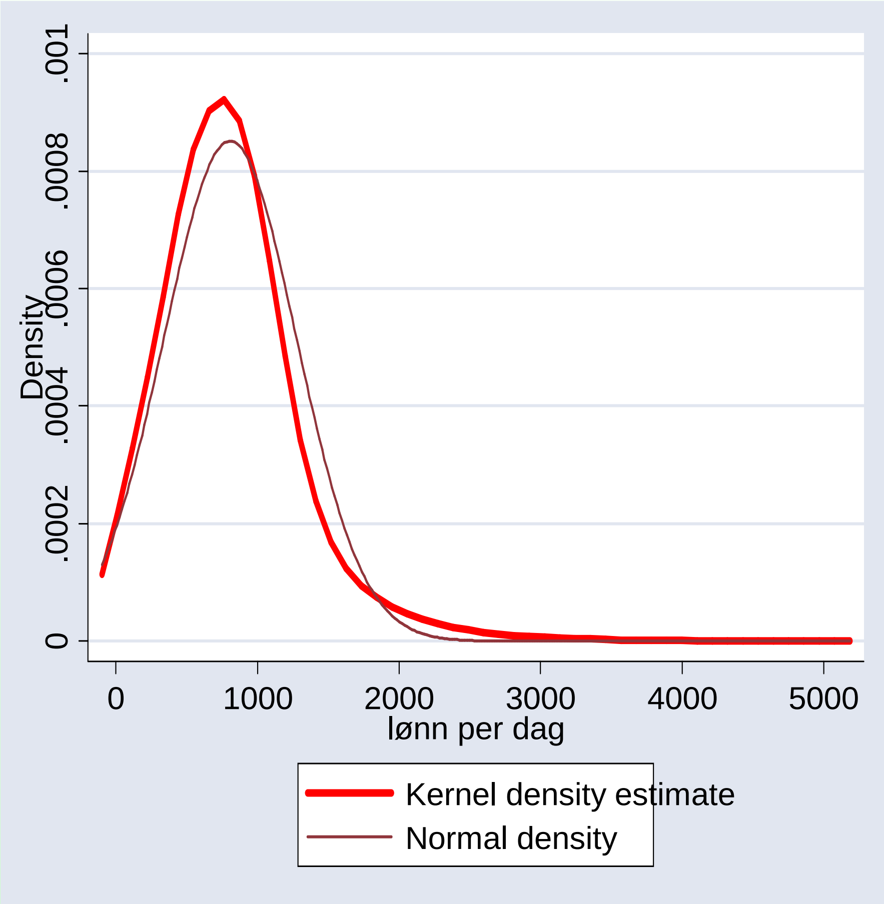
$$H(w) = \frac{q+\lambda_e}{\lambda_e}\left[1 - \sqrt{\frac{(y-x)}{(y-w)}}\right]$$

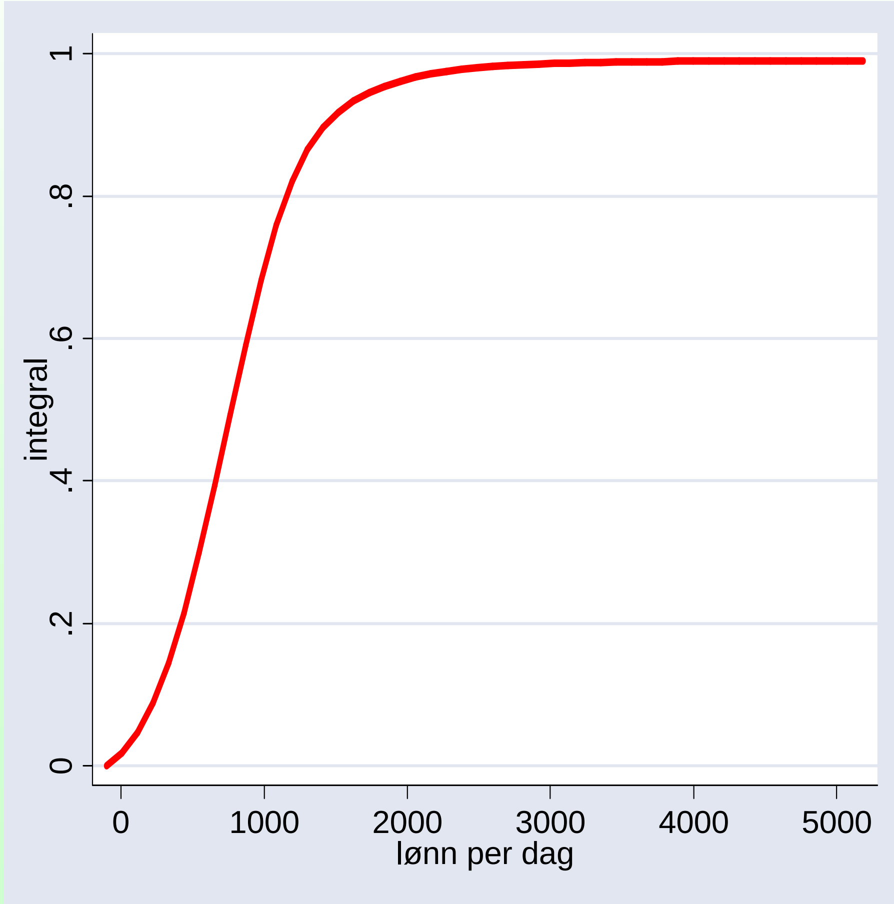
  - This wage distribution, however, *increases* in wages!

$$H'(w) = \frac{q+\lambda_e}{2\lambda_e}\frac{1}{\sqrt{y-x}\sqrt{y-w}} > 0$$

  - How do we reconcile this with the empirical wage distributions?

Wage distribution 2003 (1%-random sample)





The insurance and incentive trade-off in the unemployment benefit system

- Classical moral hazard problem (pricipal-agent): often you do not know which job seekers whom actually search for jobs.

- Assume that the principal (public authorities) is risk-neutral and the agent (the unemployed worker) is risk averse.

- If search effort verifiable, then there is no need to give the agent incentive to find work (contract upon effort), and the optimal contract completely insures the agent against fluctuations in income. Fixed benefits and transfers.

- Non-verifiable search effort seriously complicates matters: 1)participation and 2) incentive constraints. Benefits level and transfers vary during the unemployment spell.